MATH 611

Homework 6

1. The following data are observations from the random variable , where W follows a Bernoulli (p=0.7) distribution. The variables: . W is independent of and W is independent of . Code the EM algorithm for 12 iterations and present your outcomes in 3 columns, for the first and second mean and the mixture probability W respectively.

169.14353 135.73850 102.46566 80.91151 148.45425 144.68948 106.56257 104.83559

94.81216 109.47048 95.94150 123.84673 87.18401 104.73420 111.94364 119.69467

151.77627 81.80692 116.58660 98.28933

1. Consider the random variable Y, where where both and are standard normal independent random variables and .
   1. Show analytically that the variable is distributed as .
   2. Use some numerical approach to derive the values of the MLE for the unknown parameters and .
   3. Demonstrate an EM algorithm that would approximate the values of the unknown parameters and .
2. An office has three machines that each break with probability 0.1 each day, but when there is at least one broken, then there is probability 0.5 that the repairman can fix one of them for use the next day. If we ignore the possibility of two machines breaking on the same day
   1. Determine the transition probability matrix that models the number of working machines.
   2. Using a recursive formula, derive the stationary distribution.
3. In the Ehrenfest chain example presented in class, show that the binomial stationary model satisfies the detailed balanced condition.